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Vector Functions and Space Curves

Name: ____ Date: __________________

Concepts

- Parametric Curves in the Plane and in Space
- Vector Functions and Their Limits
- Derivatives and Integrals of Vector Functions
- Tangent Lines to a Space Curve (theory)
- Arc Length of a Space Curve
- Curvature (theory)
- Velocity and Acceleration Along a Curve (theory)

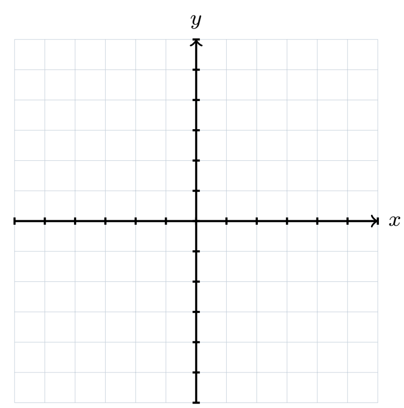
Parametric Curves in the Plane and in Space

1. A plane curve is given by $x = t^2 - 3$, $y = 2t + 1$ for $-2 \le t \le 1$.

a) Eliminate the parameter to obtain a Cartesian equation, and name the kind of curve it is.

b) Give the initial point and the terminal point, and sketch the curve on the grid with an arrow showing the direction of increasing t. Each square of the grid is one unit, and the axes cross at the origin.

c) Find every point at which the curve crosses the y-axis.



2. The cylinder $x^2 + y^2 = 9$ and the plane $y + z = 5$ meet in a closed curve C.

a) Find parametric equations for C, stating the parameter interval that traces it exactly once.

b) Find the highest and the lowest point of C.

Vector Functions and Their Limits

3. Let $\vec{r}(t) = \left(\ln(t+1),\ \sqrt{9-t^2},\ \dfrac{t}{t-2}\right)$.

a) Find the domain of $\vec{r}$, in interval notation.

b) Evaluate $\lim_{t\to 0} \vec{r}(t)$, giving the reason your method is valid.

4. Let $\vec{r}(t) = \left(\dfrac{t^2+3t-10}{t-2},\ \dfrac{\sin(\pi t)}{t-2},\ \sqrt{t+2}\right)$ for $t \geq -2$, $t \neq 2$.

a) Evaluate $\lim_{t\to 2} \vec{r}(t)$.

b) What value must be assigned to $\vec{r}(2)$ for $\vec{r}$ to be continuous at $t = 2$?

Derivatives and Integrals of Vector Functions

5. Let $\vec{r}(t) = \left(t\,e^{2t},\ \ln\!\left(1+t^2\right),\ \sin 3t\right)$. Find $\vec{r}\,'(t)$, then $\vec{r}\,'(0)$, then a unit vector tangent to the curve at the point where $t = 0$.

6. Evaluate $\displaystyle\int_0^1 \left(t\,e^t,\ \frac{2t}{1+t^2},\ \frac{1}{1+t^2}\right) dt$, giving an exact answer.

Tangent Lines to a Space Curve

7. Find parametric equations of the tangent line to the curve $\vec{r}(t) = \left(t^3 - t,\ 2t^2,\ e^{t-1}\right)$ at the point $(0, 2, 1)$.

Arc Length of a Space Curve

8. A heating coil follows the curve $\vec{r}(t) = (3\cos 2t,\ 3\sin 2t,\ 8t)$ for $0 \le t \le \pi$, with coordinates in centimetres. Find the exact length of wire in the coil.

9. Find the exact length of the curve $\vec{r}(t) = \left(5t^3 - 15t,\ 9t^2,\ 12t^2\right)$ for $0 \le t \le 2$.

Curvature

10. The curvature of a space curve is $\kappa = \dfrac{\|\vec{r}\,'(t) \times \vec{r}\,''(t)\|}{\|\vec{r}\,'(t)\|^3}$. Find the curvature of $\vec{r}(t) = \left(t^3,\ 4t,\ t^2 - 2t\right)$ at the point $(1, 4, -1)$.

Velocity and Acceleration Along a Curve

11. A radio-controlled glider has acceleration $\vec{a}(t) = (6t,\ 0,\ -2)$ m/s^2 at time t seconds, $t \geq 0$. At $t = 0$ its velocity is $(1, 2, 4)$ m/s and its position is $(0, 0, 3)$ m.

a) Find its velocity $\vec{v}(t)$ and its position $\vec{r}(t)$.

b) Find its speed at $t = 1$.

c) Find the position of the glider at the instant it is highest (largest z).

Synthesis — drawing on several topics in this unit

12. Let $\vec{r}(t) = \left(e^t + e^{-t},\ 2\sin t,\ -2\cos t\right)$.

a) Find $\vec{r}\,'(t)$, and parametric equations of the tangent line at the point $(2, 0, -2)$.

b) Show that $\|\vec{r}\,'(t)\| = e^t + e^{-t}$.

c) Find the exact length of the curve for $0 \leq t \leq \ln 3$.