

Functions of Several Variables

Name: _____

Date: _____

Concepts

- Cylinders and Quadric Surfaces (theory)
 - Limits Along Paths
 - Domain and Range of a Function of Two Variables
 - Continuity of a Function of Several Variables (theory)
 - Level Curves and Level Surfaces
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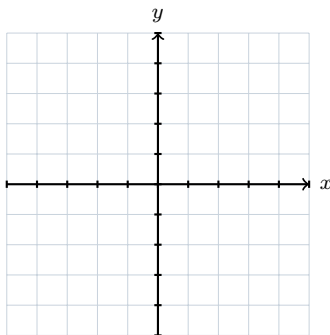
Cylinders and Quadric Surfaces

1. Consider the surface in space with equation $x^2 + \frac{y^2}{4} - z^2 = 1$.
 - a) Find the trace of the surface in the horizontal plane $z = k$, and say what kind of curve it is for every value of k .
 - b) Find the traces in the coordinate planes $x = 0$ and $y = 0$, and name each curve.
 - c) Name the surface, and state which coordinate axis is its axis.
 - d) A second surface has equation $z = 4 - y^2$, with no x in it. Describe this surface, and explain why it is called a cylinder although no slice of it is a circle.

Domain and Range of a Function of Two Variables

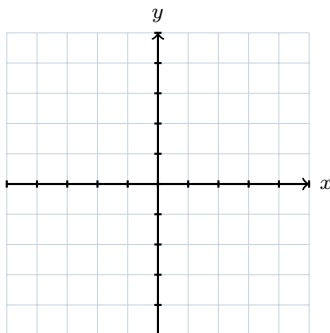
2. Let $f(x, y) = \sqrt{16 - x^2 - 4y^2}$.

- a) Evaluate $f(2, -1)$ exactly.
- b) Write the domain of f as a set defined by an inequality, describe it in words, and sketch it on the grid (each square is one unit). Show clearly whether the boundary belongs to the domain.
- c) Find the range of f , with a reason.



3. Let $g(x, y) = \frac{\ln(x + y - 2)}{\sqrt{16 - x^2 - y^2}}$.

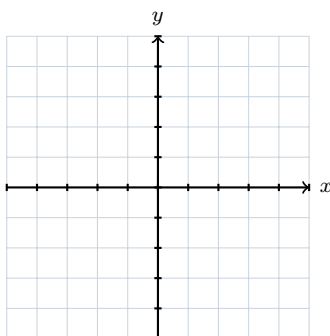
- State the two conditions a point (x, y) must satisfy to be in the domain of g , and say why each inequality is strict.
- Sketch the domain on the grid (each square is one unit), using dashed curves for boundaries that are not included.
- Decide which of the points $A(2, 1)$, $B(4, 0)$, $C(0, 2)$ and $D(3, 3)$ are in the domain, and evaluate g at each one that is.



Level Curves and Level Surfaces

4. Let $f(x, y) = y^2 - x^2$.

- a) Find the equations of the level curves $f(x, y) = k$ for $k = -4$, $k = 0$ and $k = 4$, and name each curve.
- b) Sketch the three level curves on the grid (each square is one unit), labelling each with its value of k .
- c) The point $(1, 3)$ lies on none of the three curves above. Find the value of k for which the level curve $f(x, y) = k$ passes through $(1, 3)$, and write the equation of that curve.
- d) Name the surface $z = y^2 - x^2$.



5. Let $F(x, y, z) = x^2 + y^2 + z^2 - 2z$.

- a) Describe the level surface $F(x, y, z) = k$ for $k = 3$, for $k = -1$ and for $k = -2$.
- b) Use part (a) to state the range of F .
- c) Find the level surface of F that passes through the point $(2, 1, 3)$, and describe it.

Limits Along Paths

6. Work with each limit as directed.

- a) Evaluate $\lim_{(x,y) \rightarrow (2,-1)} \frac{x^2y + 6}{x - y}$, and say why direct substitution is allowed.
- b) Show that $\lim_{(x,y) \rightarrow (0,0)} \frac{3x^2 - y^2}{x^2 + 2y^2}$ does not exist, by finding the limit along each coordinate axis.

7. Both limits below give $\frac{0}{0}$ on direct substitution.

- a) Evaluate $\lim_{(x,y) \rightarrow (4,1)} \frac{x - 4y}{\sqrt{x} - 2\sqrt{y}}$, where (x, y) is restricted to points with $x > 0$, $y > 0$ and $x \neq 4y$.
- b) Let $w(x, y) = \frac{4(x-1)y}{(x-1)^2 + 3y^2}$. Find the limit of w as $(x, y) \rightarrow (1, 0)$ along the line $y = m(x-1)$. Then decide whether $\lim_{(x,y) \rightarrow (1,0)} w(x, y)$ exists.

Continuity of a Function of Several Variables

8. Decide continuity from the way each function is built, naming the fact you use.

- a) Find the largest set on which $f(x, y) = \frac{x+y}{x^2-y}$ is continuous, and describe the set of points where it is not.
- b) Find the largest set on which $g(x, y) = \ln(1+x-y^2)$ is continuous, and describe it in words.
- c) Use continuity to evaluate $\lim_{(x,y) \rightarrow (3,-1)} g(x, y)$.

Synthesis — drawing on several topics in this unit

9. Let $f(x, y) = \sqrt{\frac{x^2}{4} + y^2} - 1$.

- a) Find the domain of f and describe it in words. Is f continuous at every point of its domain?
- b) Find the equations of the level curves $f = 0$ and $f = \sqrt{3}$, and name them.
- c) The graph of f is the set of points with $z = f(x, y)$. Show that every point of the graph lies on a quadric surface, name that quadric and its axis, and say which part of the quadric the graph is.
- d) State the range of f , with a reason.

Want the harder problems and the worked solutions?

The *Challenge Problems* set and the *Answer Keys* for Functions of Several Variables — with the reasoning written out in full — are available at tutorinmontreal.ca.