University Calculus III · Practice Worksheet · tutorinmontreal.ca

Partial Derivatives

Name: ____ Date: __________________

Concepts

- First-Order Partial Derivatives
- Higher-Order and Mixed Partial Derivatives
- Tangent Planes to a Graph
- Linear Approximation and Differentials
- The Chain Rule for Several Variables
- Implicit Differentiation with Partial Derivatives (theory)

First-Order Partial Derivatives

1. Find every first-order partial derivative of each function.

a) $f(x, y) = 3x^4y^2 - 5xy^3 + 2y$

b) $g(x, y) = x\, e^{xy^2}$

c) $h(x, y, z) = z \ln(x^2 + yz)$, on the set where $x^2 + yz > 0$

2. The temperature of a thin metal plate at the point (x, y) is

$$T(x,y) = \frac{120}{2 + x^2 + 3y^2}\ {}^\circ\mathrm{C},$$

with x and y measured in centimetres.

a) Find $T_x(1,1)$ and $T_y(1,1)$.

b) State, with units, what each of the two numbers says about the plate.

c) A probe at $(1,1)$ can be moved a short distance parallel to the x-axis or the same distance parallel to the y-axis. Which move changes its reading more, and by about what factor?

Higher-Order and Mixed Partial Derivatives

3. Let $f(x,y) = x^2 \sin(3y) + y\, e^{2x}$. Find f_{xx}, f_{xy}, f_{yx} and f_{yy}, computing f_{xy} and f_{yx} separately, and state the relation between them that your answers confirm.

4. Let
$$g(x,y) = x^2y^3 + \frac{\sin(y^2)}{1+y^4}.$$
Find g_{yyx}, that is $\dfrac{\partial^3 g}{\partial x\,\partial y\,\partial y}$, with as little work as you can, and justify the order in which you chose to differentiate.

Tangent Planes to a Graph

5. Find an equation of the tangent plane to the surface $z = x^2y - y^3 + 2x$ at the point where $(x, y) = (2, 1)$. Give your answer in the form $ax + by + cz = d$.

6. Find every point on the surface $z = x^2 + xy + y^2$ at which the tangent plane is parallel to the plane $6x - 2z = 5$, and give an equation of the tangent plane at each such point.

Linear Approximation and Differentials

7. Let $f(x, y) = \sqrt{2x + y^2}$.

a) Find the linearization $L(x, y)$ of f at $(4, 1)$.

b) Use it to approximate $f(4.06,\ 0.97)$.

8. The density of a solid ball of mass m and radius r is $\delta = \dfrac{3m}{4\pi r^3}$. The mass of a steel ball is measured with an error of at most 2% and its radius with an error of at most 1%. Use differentials to estimate the maximum percentage error in the computed density, and say which of the two measurements contributes more to it.

The Chain Rule for Several Variables

9. Let $z = x^2y - 3xy^2$, where $x = 2t + 1$ and $y = t^2$. Use the chain rule to find $\dfrac{dz}{dt}$ at $t = 1$, without first writing z as a function of t.

10. Let $z = x^2y + x\,e^y$, where $x = st$ and $y = s^2 - 4t^2$.

a) Draw the tree diagram linking z to s and t, and write the chain rule for $\dfrac{\partial z}{\partial s}$ and for $\dfrac{\partial z}{\partial t}$.

b) Evaluate both at $(s, t) = (2, 1)$.

Implicit Differentiation with Partial Derivatives

11. The equation $x^2z + yz^3 - 2xy = -1$ defines z implicitly as a differentiable function of x and y near the point $(1, 2, 1)$.

a) Verify that $(1, 2, 1)$ satisfies the equation.

b) Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ in terms of x, y and z, and evaluate them at $(1, 2, 1)$.

Synthesis — drawing on several topics in this unit

12. Let $f(x, y) = y \ln x + xy^2$ for $x > 0$.

a) Find f_x and f_y, and verify that $f_{xy} = f_{yx}$.

b) Find an equation of the tangent plane to the surface $z = f(x, y)$ at the point where $(x, y) = (1, 2)$.

c) Use your answer to (b) to estimate $f(1.03,\ 1.98)$.

Want the harder problems and the worked solutions?

The *Challenge Problems* set and the *Answer Keys* for Partial Derivatives — with the reasoning written out in full — are available at **tutorinmontreal.ca**.