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Nonhomogeneous Equations and Variable Coefficients

Name: _____

Date: _____

Concepts

- Undetermined Coefficients and the Form of the Trial Solution
- Undetermined Coefficients When the Trial Solution Duplicates a Term
- Variation of Parameters
- Cauchy-Euler Equations
- Reduction of Order from One Known Solution (theory)

Undetermined Coefficients and the Form of the Trial Solution

1. Here $y = y(x)$.

a) Find the general solution of $y'' + 3y' + 2y = 4x^2$ by undetermined coefficients.

b) For the same left-hand side $y'' + 3y' + 2y$, write down the form of a trial particular solution for each right-hand side below. Do not evaluate the coefficients.

(i) xe^{3x} (ii) $e^x \cos 2x$ (iii) $3 - \sin x$

2. Solve the initial value problem $y'' - 4y' + 3y = 10 \cos t$, $y(0) = 1$, $y'(0) = 0$, where $y = y(t)$.

Undetermined Coefficients When the Trial Solution Duplicates a Term

3. Find the general solution of each equation, where $y = y(x)$.

a) $y'' + 2y' - 3y = 8e^x$

b) $y'' + 4y' = 8x + 2$

4. Solve the initial value problem $y'' - 6y' + 9y = 6e^{3t}$, $y(0) = 1$, $y'(0) = 2$, where $y = y(t)$.

Variation of Parameters

5. Use variation of parameters to find the general solution of

$$y'' - 4y' + 4y = \frac{e^{2x}}{x^2}, \quad x > 0,$$

where $y = y(x)$.

6. Use variation of parameters to find the general solution of

$$y'' + 9y = 9 \sec^2 3x$$

on the interval $(-\frac{\pi}{6}, \frac{\pi}{6})$, where $y = y(x)$.

Cauchy-Euler Equations

7. Find the general solution of each equation on $(0, \infty)$, where $y = y(x)$.

a) $x^2y'' - 2xy' - 4y = 0$

b) $x^2y'' + 5xy' + 4y = 0$

c) $x^2y'' - xy' + 5y = 0$

8. Solve the initial value problem $x^2y'' + 3xy' - 3y = 0$, $y(1) = 4$, $y'(1) = 0$, where $y = y(x)$, and give the largest interval on which the solution is valid.

Reduction of Order from One Known Solution

9. The function $y_1 = e^x$ is a solution of

$$xy'' - (x+1)y' + y = 0, \quad x > 0,$$

where $y = y(x)$. Use reduction of order, with $y = u(x)e^x$, to find a second, linearly independent solution, and write the general solution.

Synthesis — drawing on several topics in this unit

10. Consider $y'' + 4y = 4x - 8 \sin 2x$, where $y = y(x)$.

- a) Write the form of a trial particular solution, explaining which part of it had to be modified and why.
- b) Solve the initial value problem with $y(0) = 0$ and $y'(0) = 0$.

Want the harder problems and the worked solutions?

The *Challenge Problems* set and the *Answer Keys* for Nonhomogeneous Equations and Variable Coefficients — with the reasoning written out in full — are available at tutorinmontreal.ca.