University Discrete Mathematics · Practice Worksheet · tutorinmontreal.ca

Propositional and Predicate Logic

Name: ______________________ Date: ____________

Concepts

- Propositions and Logical Connectives
- Truth Tables (theory)
- Conditional Statements, Converse and Contrapositive
- Logical Equivalences and De Morgan's Laws
- Predicates and Quantifiers
- Negating Quantified Statements
- Nested Quantifiers (theory)
- Rules of Inference and Valid Arguments

Propositions and Logical Connectives

1. Decide which of the following are propositions. For each one that is, give its truth value; for each one that is not, say why.

a) $2^{10} < 1000$.

b) $x + 3 = 10$.

c) Every integer greater than 1 has a prime divisor.

d) Is 91 a prime number?

e) 91 is a prime number.

f) Close the window before you leave.

2. Let p be "the train is late", q be "Mila takes the bus" and r be "Mila arrives before 9:00". Write each sentence in symbols.

a) The train is late and Mila does not take the bus.

b) Mila arrives before 9:00 only if she takes the bus.

c) Mila takes the bus unless the train is late.

d) Either the train is late or Mila arrives before 9:00, but not both.

Then write $(q \wedge \neg p) \to r$ as an English sentence.

Truth Tables

3. Construct the truth table of $(p \vee q) \to (\neg p \wedge r)$ and classify the proposition as a tautology, a contradiction or a contingency.

Conditional Statements, Converse and Contrapositive

4. Consider the statement "For every integer n, if n is divisible by 6, then n is divisible by 3." Write the converse, the inverse and the contrapositive of the conditional inside it (each still for every integer n), and decide which of the four statements are true. Give a counterexample for each false one.

5. Suppose p is true, q is false and r is true. Find the truth value of each proposition.

a) $(p \to q) \to r$

b) $(q \to p) \land (r \to q)$

c) $\neg p \to (q \leftrightarrow r)$

Logical Equivalences and De Morgan's Laws

6.

a) Use De Morgan's laws to write the negation of "The file is encrypted and the file is backed up" and of "Jonah is in the lab or Jonah is at the library", without the words "it is not the case that".

b) Use De Morgan's laws and double negation to rewrite $\neg\big(\neg p \land (q \lor \neg r)\big)$ as an equivalent proposition in which $\neg$ is applied only to single variables.

7. Use a truth table to decide whether $p \to (q \to r)$ and $(p \to q) \to r$ are logically equivalent. If they are not, list every row in which they differ.

Predicates and Quantifiers

8. The domain is the climbers at a climbing gym. Let $L(x)$ be "x climbs lead" and $C(x)$ be "x is certified to belay". Write each sentence in symbols.

a) Every climber who climbs lead is certified to belay.

b) Some climber who is certified to belay does not climb lead.

c) No climber climbs lead.

Then write $\exists x\,(L(x) \wedge \neg C(x))$ as an English sentence.

9. Let $P(x)$ be the predicate "$x^2 > 2x$".

a) Find the truth values of $P(0)$, $P(3)$, $P(-1)$ and $P(2)$.

b) Find the truth values of $\forall x \in \mathbb{Z},\ P(x)$ and of $\exists x \in \mathbb{Z},\ P(x)$.

c) Find the truth value of $\forall x \in D,\ P(x)$, where $D = \{x \in \mathbb{Z} \mid x \geq 3\}$, and justify it.

Negating Quantified Statements

10. Write the negation of each statement so that no $\neg$ stands in front of a quantifier or a bracket. Then decide whether the original or its negation is true, with a one-line reason.

a) $\forall x \in \mathbb{R},\ (x > 0 \rightarrow x^2 > x)$

b) $\exists n \in \mathbb{Z},\ n^2 = n + 1$

c) $\exists x \in \mathbb{N},\ (x \text{ is even} \wedge x \text{ is prime})$, where $\mathbb{N} = \{0, 1, 2, \dots\}$

11. Write the negation of each sentence in natural English, without the words "it is not the case that".

a) Every bus on this route arrives on time.

b) Some password in the file is shorter than 8 characters and contains no digit.

c) No student in the class submitted the assignment late.

Nested Quantifiers

12. The domain of every variable is $\mathbb{Z}$. Find the truth value of each statement, with a one-line reason.

a) $\forall x\, \exists y,\ x + y = 0$

b) $\exists y\, \forall x,\ x + y = x$

c) $\forall x\, \exists y,\ 2y = x$

d) $\exists x\, \forall y,\ x \leq y$

Rules of Inference and Valid Arguments

13. For each argument, write its form in symbols and name the rule of inference it uses, or say that it is not valid.

a) If the reactor temperature exceeds 300 degrees, the alarm sounds. The alarm did not sound. Therefore the temperature did not exceed 300 degrees.

b) Kofi is in Toronto or Kofi is in Ottawa. Kofi is not in Toronto. Therefore Kofi is in Ottawa.

c) If an integer is divisible by 9, it is divisible by 3. The integer 51 is divisible by 3. Therefore 51 is divisible by 9.

d) If the meeting runs late, Inès misses the ferry. If Inès misses the ferry, she takes a taxi. Therefore, if the meeting runs late, Inès takes a taxi.

14. Decide whether the argument with premises $p \to q$, $q \vee r$, $\neg r$ and conclusion p is valid. If it is not, give an assignment of truth values that shows it.

Synthesis — drawing on several topics in this unit

15. The domain is $\mathbb{Z}^+$, the positive integers. Let $P(n)$ be "n is prime" and let S be the statement

$$\forall n \in \mathbb{Z}^+,\ \big(P(n) \to P(n+2)\big).$$

a) Write the negation of S with no $\neg$ in front of a quantifier or a bracket.

b) Decide which of S and its negation is true, and give a witness or a counterexample.

c) Write the contrapositive of the conditional inside S as an English sentence.