University Introductory Statistics · Practice Worksheet · tutorinmontreal.ca

Probability Rules and Conditional Probability

Name: ______________________________ Date: ______________

Concepts

- Sample Spaces, Events and Venn Diagrams (theory)
- The Addition Rule and Complements
- Counting with Permutations and Combinations (theory)
- Conditional Probability and the Multiplication Rule
- Independent Events
- Tree Diagrams, Total Probability and Bayes' Theorem

Sample Spaces, Events and Venn Diagrams

1. A trail camera is left in a forest for three nights. Each night it either records a fox (D) or does not (N), so an outcome is a string such as DND (fox on nights 1 and 3 only). Let A be the event that a fox is recorded on at least two of the nights, and B the event that a fox is recorded on the first night.

a) List the sample space.

b) List the outcomes in A, B, $A \cap B$, $A \cup B$ and A^c.

c) Are A and B mutually exclusive? Justify.

d) Assuming the eight outcomes are equally likely, find $P(A \cup B)$ and $P(A^c)$.

The Addition Rule and Complements

2. A seed mix for a wildflower garden produces red, yellow, white or purple flowers. For a seed chosen at random, $P(\text{red}) = 0.35$, $P(\text{yellow}) = 0.25$ and $P(\text{white}) = 0.15$; each seed produces exactly one colour.

a) Find $P(\text{purple})$.

b) Find the probability that the flower is red or yellow.

c) Find the probability that it is not white.

d) Find the probability that it is neither red nor white.

3. In a cycling club, a member chosen at random owns a road bike with probability 0.55 and a mountain bike with probability 0.40, and owns at least one of the two with probability 0.80.

a) Find the probability that the member owns both kinds of bike.

b) Find the probability that the member owns a road bike but no mountain bike.

c) Find the probability that the member owns neither kind, and the probability that the member owns exactly one kind.

d) Are the events “owns a road bike” and “owns a mountain bike” mutually exclusive? Justify.

Counting with Permutations and Combinations

4. An ecology class has 11 sampling plots along a river, 4 of them in wetland and 7 on dry ground. A team chooses 3 plots at random to survey, every set of 3 being equally likely.

a) How many different sets of 3 plots are possible?

b) Find the probability that exactly 2 of the chosen plots are wetland.

c) Find the probability that at least one chosen plot is wetland.

d) The team will survey its 3 plots in a fixed order: morning, midday and afternoon. How many different schedules (which plot at which time) can be made from the 11 plots?

Conditional Probability and the Multiplication Rule

5. A regional transit authority classified 400 surveyed riders by fare type and by the purpose of their trip.

	Work	School	Other	Total
Monthly pass	126	54	30	210
Single ticket	42	36	112	190
Total	168	90	142	400

One surveyed rider is chosen at random. Find

a) $P(\text{monthly pass})$;

b) $P(\text{work} \mid \text{monthly pass})$;

c) $P(\text{monthly pass} \mid \text{work})$;

d) $P(\text{monthly pass} \cap \text{school})$ and $P(\text{other} \mid \text{single ticket})$.

6.

a) A literary magazine passes 40% of submitted stories through its first editorial screen. Of the stories that pass, 35% are accepted by the reviewers, and of the accepted stories, 50% are printed in the next issue. Find the probability that a submitted story is printed in the next issue, and the probability that it passes the screen but is then turned down by the reviewers.

b) A tray holds 10 pastries that look identical, 3 of which have a nut filling. A guest takes 3 pastries at random, one after another. Use the multiplication rule to find the probability that none of the three has a nut filling, and the probability that the first two both do.

Independent Events

7. For an order chosen at random at an online shop, let G be the event that the order is gift-wrapped and X the event that it is sent by express shipping. The shop's records give $P(G) = 0.20$, $P(X) = 0.35$ and $P(G \cap X) = 0.07$.

a) Decide whether G and X are independent.

b) Find $P(G \mid X)$ and say what it tells the shop.

c) Find $P(G \cup X)$.

d) Are G and X mutually exclusive? Justify.

8. A warehouse is protected at night by a motion sensor, a camera and a guard patrol, which detect an intruder independently of one another with probabilities 0.9, 0.8 and 0.5. Find the probability that an intruder is detected

a) by all three;

b) by none of them;

c) by at least one of them;

d) by the motion sensor only.

Tree Diagrams, Total Probability and Bayes' Theorem

9. A language-learning app gets its new users from three channels: 50% from search advertising, 30% from referrals by friends and 20% from social media. The probability that a new user is still active after 30 days is 0.20 for search users, 0.40 for referred users and 0.10 for social-media users.

a) Draw a tree diagram with every branch labelled.

b) Find the probability that a new user is still active after 30 days.

c) An active user is chosen at random. Find the probability that the user came from a referral.

d) A user who is *not* active is chosen at random. Find the probability that the user came from search advertising.

10. At an orchard's packing line, 8% of apples are bruised. An optical scanner flags a bruised apple with probability 0.95 and flags a sound apple with probability 0.05.

a) Find the probability that a randomly chosen apple is flagged.

b) Find the probability that a flagged apple is bruised.

c) Find the probability that an apple that is not flagged is bruised.

Synthesis — drawing on several topics in this unit

11. A laboratory keeps two racks of sample vials. Rack 1 holds 5 vials, 2 of them contaminated; rack 2 holds 6 vials, 1 of them contaminated. A technician rolls a fair die and uses rack 1 on a roll of 1 or 2 and rack 2 otherwise, then takes 2 vials at random from that rack.

a) For each rack, find the probability that both vials taken are clean.

b) Find the probability that both vials taken are clean.

c) Given that both vials are clean, find the probability that they came from rack 1.