

Discrete Random Variables

Name: _____

Date: _____

Concepts

- Probability Mass Functions and Cumulative Distributions (theory)
 - Expected Value of a Discrete Random Variable
 - Variance and Linear Transformations of a Random Variable
 - Bernoulli Trials and the Binomial Formula
 - Binomial Tables, Mean and Variance
 - The Poisson Distribution (theory)
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Probability Mass Functions and Cumulative Distributions

1. A driving school records X , the number of attempts a student needs to pass the road test. Its instructors model the distribution by

$$P(X = x) = k(5 - x), \quad x = 1, 2, 3, 4,$$

and $P(X = x) = 0$ for every other x .

- Find the constant k and write the probability mass function as a table.
- Write the cumulative distribution function $F(x) = P(X \leq x)$ for *every* real number x .
- Use F to find $P(X \leq 2.5)$, $P(1 < X \leq 3)$ and $P(X \geq 2)$.

Expected Value of a Discrete Random Variable

2. An ice-cream stand sells cones of 1 to 4 scoops. For a cone chosen at random, the number of scoops X has the distribution below, and the stand's records show that the mean number of scoops per cone is $E[X] = 2.5$.

x	1	2	3	4
$P(X = x)$	a	0.3	b	0.2

- a) Find a and b .
- b) Find the probability that a cone has more scoops than the mean.

3. At opening time, the number X of bicycles out of service at a bike-share station has the distribution

x	0	1	2	3	4
$P(X = x)$	0.35	0.30	0.20	0.10	0.05

If at least one bicycle is out of service, the operator pays a technician a call-out fee of \$30 plus \$12 per bicycle repaired; if none is, it pays nothing.

- a) Find $E[X]$.
- b) Find the expected daily repair cost.

Variance and Linear Transformations of a Random Variable

4. The number X of written complaints a hotel front desk logs in one evening shift has the distribution

x	0	1	2	3
$P(X = x)$	0.1	0.4	0.3	0.2

Find $E[X]$, $\text{Var}(X)$ and $\text{SD}(X)$. Compute the variance twice: from the definition $E[(X - \mu)^2]$, and from the shortcut $E[X^2] - \mu^2$.

5. A ski-rental shop rents X snowboards in a weekday hour, where $E[X] = 6$ and $\text{SD}(X) = 2$. Each rental brings in \$25, and the hour's staffing costs \$40, so the hour's net revenue is $R = 25X - 40$ dollars.

a) Find $E[R]$, $\text{Var}(R)$ and $\text{SD}(R)$.

b) Find the mean and the standard deviation of $Z = \frac{X - 6}{2}$.

Bernoulli Trials and the Binomial Formula

6. For each situation, decide whether X has a binomial distribution, exactly or to a good approximation (say which). If it does, give n and p ; if it does not, name a condition of the binomial model that fails.

- a) A student guesses at random on all 8 questions of a quiz, each with 4 options of which one is correct; X is the number of correct answers.
- b) From a box of 10 phone chargers, 4 of them faulty, a technician takes 3 without replacement; X is the number of faulty chargers taken.
- c) A player rolls a fair die until a six appears; X is the number of rolls.
- d) In a city of 2 million adults, 30% cycle to work. A survey chooses 20 adults at random; X is the number of them who cycle to work.

7. A gardener plants 6 seeds of a variety for which each seed, independently, fails to germinate with probability 0.3. Let X be the number of seeds that fail. You may use $0.7^4 = 0.2401$, $0.7^5 = 0.16807$ and $0.7^6 = 0.117649$.

- a) Find $P(X = 2)$.
- b) Find the probability that at most one seed fails.
- c) Find the probability that at least two seeds fail.

Binomial Tables, Mean and Variance

8. An online shop finds that each order, independently, is returned with probability 0.3. Let X be the number of returns among the next 12 orders, so $X \sim \text{Bin}(12, 0.3)$. Its cumulative probabilities are

k	0	1	2	3	4	5	6	7	8
$P(X \leq k)$	0.0138	0.0850	0.2528	0.4925	0.7237	0.8822	0.9614	0.9905	0.9983

Use the table to find

- a) $P(X \leq 4)$;
- b) $P(X = 4)$;
- c) $P(X > 5)$;
- d) $P(3 \leq X \leq 7)$;
- e) $P(X < 3)$.

9.

- a) A courier knows that 12% of its parcels, independently, need a second delivery attempt. For a driver's round of 50 parcels, let X be the number needing a second attempt. Find $E[X]$, $\text{Var}(X)$ and $\text{SD}(X)$.
- b) A binomial variable Y has mean 6 and variance 4.2. Find its n and p .

The Poisson Distribution

10. Calls reach a small software firm's help line at an average rate of 3 per hour, and the number of calls in an hour is modelled as $X \sim \text{Poisson}(3)$, so that $P(X = x) = \frac{e^{-3}3^x}{x!}$ for $x = 0, 1, 2, \dots$. You may use $e^{-3} = 0.049787$ and $e^{-1} = 0.367879$.

- a) Find $P(X = 2)$ and $P(X \leq 2)$.
- b) State the mean and the standard deviation of X .
- c) Find the probability that no call arrives during a 20-minute coffee break, and the probability that at least one does.

Synthesis — drawing on several topics in this unit

11. An airport shuttle van has 10 passenger seats. Because some passengers do not turn up, the company accepts 11 bookings for each trip. Each booked passenger turns up with probability 0.9, independently of the others. Let X be the number who turn up. You may use $P(X = 10) = 0.3835$ and $P(X = 11) = 0.3138$.

- a) Name the distribution of X , with its parameters, and find $E[X]$ and $SD(X)$.
- b) Find the probability that the van is overbooked (a passenger who turns up has no seat).
- c) Find the probability that the van leaves with at least one empty seat.
- d) Let B be the number of passengers left without a seat. Write the distribution of B and find $E[B]$.

Want the harder problems and the worked solutions?

The *Challenge Problems* set and the *Answer Keys* for Discrete Random Variables — with the reasoning written out in full — are available at tutorinmontreal.ca.