University Introductory Statistics · Practice Worksheet · tutorinmontreal.ca

Confidence Intervals

Name: ____ Date: __________________

Concepts

- Interpreting a Confidence Interval
- Large-Sample Confidence Interval for a Mean
- The t Distribution and Small-Sample Intervals for a Mean
- Confidence Interval for a Proportion
- Sample Size for a Given Margin of Error (theory)

Interpreting a Confidence Interval

1. A biologist nets 36 trout at random from a lake and reports a 90% confidence interval for the mean length of the lake's trout of $(31.2,\ 34.8)$ cm. The interval was computed as $\bar{x} \pm z_{0.05}\,\dfrac{s}{\sqrt{n}}$. Use $z_{0.05} = 1.645$ and $z_{0.025} = 1.960$.

a) Find the sample mean $\bar{x}$ and the margin of error.

b) Find the sample standard deviation s the biologist must have used.

c) Without recomputing anything else, say whether the 95% interval from the same data is wider or narrower, and why. Then compute it.

2. Twenty food-safety laboratories each draw their own random sample of jars from the same very large production run and each compute a 90% confidence interval for the mean sodium content μ of the run. The laboratories work independently.

a) How many of the twenty intervals would you expect to contain μ? How many to miss it?

b) Let X be the number of the twenty intervals that contain μ. Name the distribution of X, with its parameters, and find $E[X]$ and $\mathrm{SD}(X)$.

c) Seventeen of the intervals turn out to contain μ. Is that evidence that the laboratories did something wrong? Answer in one sentence.

Large-Sample Confidence Interval for a Mean

3. Years of testing have shown that simple reaction times on a lab's standard task have standard deviation $\sigma = 12$ ms. A random sample of 36 volunteers gives a mean reaction time of $\bar{x} = 248$ ms. Use $z_{0.05} = 1.645$ and $z_{0.025} = 1.960$.

a) State the conditions that justify a z interval here.

b) Construct a 95% confidence interval for the mean reaction time μ of the population the volunteers were drawn from.

4. A city's transport department asks a random sample of 49 residents who cycle to work how far they ride each way. The sample mean is $\bar{x} = 7.4$ km and the sample standard deviation is $s = 2.8$ km. Use $z_{0.05} = 1.645$, $z_{0.025} = 1.960$ and $z_{0.005} = 2.576$.

a) Construct a 90% and a 99% confidence interval for the mean one-way distance μ ridden by the city's cycling commuters. State the conditions you rely on.

b) By what factor is the 99% interval wider than the 90% interval?

The t Distribution and Small-Sample Intervals for a Mean

5. A bakery weighs 10 randomly chosen loaves from one day's batch (grams):

$$494, \quad 504, \quad 498, \quad 508, \quad 500, \quad 496, \quad 502, \quad 506, \quad 492, \quad 500.$$

Loaf masses from this oven are known from long experience to be approximately normal. Use $t_{0.025,\,9} = 2.262$, $t_{0.025,\,10} = 2.228$ and $z_{0.025} = 1.960$.

a) Find $\bar{x}$ and s.

b) Construct a 95% confidence interval for the mean mass μ of the day's loaves, and state which printed value you used and why the other two are wrong here.

6. A start-up tests 16 prototype e-bike batteries, chosen at random from its first production run, and records the range of each on a full charge: $\bar{x} = 62.5$ km, $s = 8.0$ km. Ranges of batteries of this design are approximately normal. Use $t_{0.005,\,15} = 2.947$, $t_{0.005,\,16} = 2.921$, $t_{0.01,\,15} = 2.602$, $t_{0.025,\,4} = 2.776$, $t_{0.025,\,15} = 2.131$, $t_{0.025,\,29} = 2.045$ and $z_{0.025} = 1.960$.

a) Construct a 99% confidence interval for the mean range μ of the run.

b) Using only the printed values, describe how $t_{0.025,\,\nu}$ changes as the degrees of freedom ν grow, and explain why.

Confidence Interval for a Proportion

7. A transit authority asks a random sample of 625 riders whether they would use a new late-night bus route; 225 say yes. Use $z_{0.05} = 1.645$ and $z_{0.025} = 1.960$. The large-sample interval for a proportion requires a random sample with at least 10 successes and at least 10 failures.

a) Check the conditions.

b) Construct a 95% confidence interval for the proportion p of all riders who would use the route.

8. A phone repair chain inspects a random sample of 300 replacement screens from a supplier and finds 18 defective. The large-sample interval for a proportion requires a random sample with at least 10 successes and at least 10 failures. Use $z_{0.025} = 1.960$, $z_{0.01} = 2.326$ and $z_{0.005} = 2.576$.

a) Construct a 99% confidence interval for the supplier's defect rate p, after checking the conditions.

b) The supplier's contract promises a defect rate of at most 2%. Is a rate of 2% a plausible value of p, according to the interval?

Sample Size for a Given Margin of Error

9. Use $z_{0.05} = 1.645$, $z_{0.025} = 1.960$ and $z_{0.005} = 2.576$, and the formulas

$$n = \left(\frac{z_{\alpha/2}\,\sigma}{E}\right)^2 \qquad \text{and} \qquad n = \left(\frac{z_{\alpha/2}}{E}\right)^2 p^*(1-p^*),$$

where E is the desired margin of error and p^* a planning value of the proportion; with no prior estimate of the proportion, take $p^* = 0.5$. Round every sample size **up** to the next whole number.

a) A dietitian wants to estimate the mean daily sugar intake of teenagers to within 3 g with 95% confidence. A pilot study suggests $\sigma \approx 15$ g. How many teenagers should she sample?

b) A student union wants to estimate, to within 4 percentage points with 90% confidence, the proportion of students who would pay for a campus bike-share. It has no prior idea of the proportion. How many students should it survey?

Synthesis — drawing on several topics in this unit

10. A residence survey asks a random sample of 100 first-year students how many hours they slept the previous night. The sample mean is 6.8 h and the sample standard deviation is 1.2 h; 38 of the 100 students slept less than 6 h. Use $z_{0.05} = 1.645$ and $z_{0.025} = 1.960$. The large-sample interval for a proportion requires a random sample with at least 10 successes and at least 10 failures.

a) Construct a 95% confidence interval for the mean hours of sleep μ of all first-year students, stating the conditions.

b) Construct a 95% confidence interval for the proportion p of first-year students who slept less than 6 h, stating the conditions.

c) Using $\hat{p}$ as the planning value, how many students would a follow-up survey need for the interval in (b) to have a margin of error of 0.05 at 95% confidence? Use $n = \left(\dfrac{z_{\alpha/2}}{E}\right)^2 p^*(1-p^*)$ and round **up** to the next whole number.