University Introductory Statistics · Practice Worksheet · tutorinmontreal.ca

# Hypothesis Tests

Name: ______________________________ Date: ______________

## Concepts

- Hypotheses, Test Statistics and Rejection Regions
- The p-Value
- Large-Sample Test for a Mean
- The t Test for a Mean
- Test for a Proportion
- Type I and Type II Errors
- The Power of a Test (theory)

### Hypotheses, Test Statistics and Rejection Regions

**1.** For each situation, define the parameter in words, write $H_0$ and $H_a$, and say whether the test is left-tailed, right-tailed or two-tailed.

**a)** A ferry line advertises a mean crossing time of 42 minutes. Regular commuters suspect that crossings take longer than that on average.

**b)** A seed company states that 85% of its pepper seeds germinate. A market gardener suspects that the germination rate is lower.

**c)** A dairy labels its yogurt cups 175 g. An inspector wants to know whether the mean net mass of the cups differs from the label, in either direction.

**2.** A $z$ test of $H_0 : \mu = 60$ is carried out at significance level $\alpha = 0.05$. Use $z_{0.05} = 1.645$ and $z_{0.025} = 1.960$.

**a)** Give the rejection region for the test statistic $z$ under each alternative: $H_a : \mu > 60$, $H_a : \mu < 60$, $H_a : \mu \neq 60$.

**b)** The observed value of the statistic is $z = 1.80$. State the decision under each of the three alternatives.

### The p-Value

**3.** A large-sample test about the mean length of a visit to a science museum gives the statistic $z = 2.17$. Use $P(Z < 2.17) = 0.9850$.

**a)** Find the $p$-value if the alternative is right-tailed.

**b)** Find the $p$-value if the alternative is two-tailed.

**c)** For each of (a) and (b), state the decision at $\alpha = 0.05$ and at $\alpha = 0.01$.

**4.** A hotel chain measures the nightly energy use of a random sample of $n = 16$ rooms and runs a $t$ test for the mean, obtaining $t = 2.41$. Use

$$t_{0.05,\,15} = 1.753, \quad t_{0.025,\,15} = 2.131, \quad t_{0.01,\,15} = 2.602, \quad t_{0.005,\,15} = 2.947.$$

**a)** Give the $p$-value as a bracket if the alternative is right-tailed.

**b)** Give the $p$-value as a bracket if the alternative is two-tailed.

**c)** For each, state the decision at $\alpha = 0.05$ and at $\alpha = 0.01$.

### Large-Sample Test for a Mean

**5.** A coffee roaster sells bags labelled 250 g. A random sample of 64 bags has mean net mass $\bar{x} = 247.6$ g and standard deviation $s = 8$ g. Test at $\alpha = 0.05$ whether the mean net mass of all the roaster's bags differs from 250 g. Use $z_{0.05} = 1.645$, $z_{0.025} = 1.960$ and $P(Z < -2.40) = 0.0082$. Give the hypotheses, the conditions, the test statistic, the decision by critical value and by $p$-value, and the conclusion in context. (Treat $n \geq 30$ as large.)

**6.** A courier company's delivery times in a city have standard deviation $\sigma = 12$ minutes, known from years of records. After a change of warehouse, a random sample of 36 deliveries has mean time $\bar{x} = 53.1$ minutes. The previous mean was 50 minutes. Test at $\alpha = 0.05$ whether the mean delivery time has increased, in the five-part form. Use $z_{0.05} = 1.645$ and $P(Z < 1.55) = 0.9394$.

### The t Test for a Mean

**7.** A cycling club's route guide says a hill loop takes 45 minutes on average. Members suspect it takes longer. Eight randomly chosen rides took (minutes) 53, 54, 42, 51, 50, 48, 45, 49. Ride times are known to be roughly normally distributed. Test at $\alpha = 0.05$, in the five-part form, using a critical value. Use $t_{0.05,\,7} = 1.895$, $t_{0.025,\,7} = 2.365$, $t_{0.01,\,7} = 2.998$ and $t_{0.05,\,8} = 1.860$, and also give the $p$-value as a bracket.

**8.** A cold-brew coffee is labelled as containing 30 mg of caffeine per 100 mL. A lab tests a random sample of 25 bottles and finds $\bar{x} = 31.9$ mg and $s = 5.0$ mg per 100 mL; the measurements show no strong skew and no outliers. Test at $\alpha = 0.05$ whether the mean caffeine content differs from the label, in the five-part form with a $p$-value bracket. Use $t_{0.05,\,24} = 1.711$, $t_{0.025,\,24} = 2.064$ and $t_{0.025,\,25} = 2.060$.

### Test for a Proportion

**9.** A public library believes that more than 20% of its card holders now borrow e-books. In a random sample of 400 card holders, 96 borrow e-books. Test the belief at $\alpha = 0.05$ in the five-part form. Use $z_{0.05} = 1.645$ and $P(Z < 2.00) = 0.9772$. (Take the normal model for $\hat{p}$ as adequate when $np_0 \geq 10$ and $n(1 - p_0) \geq 10$.)

**10.** A town council states that 40% of households compost their food scraps. In a random survey of 600 households, 219 compost. Test at $\alpha = 0.05$ whether the true proportion differs from the council's figure, in the five-part form. Use $z_{0.025} = 1.960$ and $P(Z < -1.75) = 0.0401$. (Normal model adequate when $np_0 \geq 10$ and $n(1 - p_0) \geq 10$.)

### Type I and Type II Errors

**11.** A food inspector tests whether the mean sodium content of a brand of crackers exceeds the 150 mg per serving printed on the box, using $H_0 : \mu = 150$ and $H_a : \mu > 150$ at $\alpha = 0.05$. A rejection leads to a fine for the manufacturer.

**a)** Describe a Type I error and a Type II error in this context, with the consequence of each.

**b)** If the mean sodium content really is 150 mg, what is the probability that the inspector fines the manufacturer?

**c)** The inspector's test does not reject $H_0$. Which of the two errors could have been made?

**12.** An electronics buyer receives fuses in large lots. From each lot it tests 10 randomly chosen fuses and rejects the lot if 2 or more are defective. Let $p$ be the proportion defective in a lot, with $H_0 : p = 0.05$ (an acceptable lot) and $H_a : p > 0.05$, so that rejecting the lot is rejecting $H_0$. The number of defectives among the 10 is binomial.

**a)** Find the probability of a Type I error.

**b)** Find the probability of a Type II error if in fact $p = 0.20$.

Give probabilities to four decimal places.

### The Power of a Test

**13.** A decking manufacturer tests whether a new resin raises the mean breaking load of its boards above 500 kg. It uses $H_0 : \mu = 500$, $H_a : \mu > 500$ at $\alpha = 0.05$, with a random sample of 64 boards; the breaking loads have standard deviation $\sigma = 40$ kg. Use $z_{0.05} = 1.645$ and $P(Z < -0.755) = 0.2251$.

**a)** Find the values of $\bar{x}$ for which $H_0$ is rejected.

**b)** Find the probability of a Type II error, and the power of the test, if the true mean breaking load is 512 kg.

**c)** Without further calculation, say whether the power at $\mu = 512$ goes up or down if (i) $n$ is increased to 100; (ii) $\alpha$ is lowered to 0.01; (iii) the power is instead computed at $\mu = 520$. Give a reason for each.

**Synthesis — drawing on several topics in this unit**

**14.** A water treatment plant must keep the mean dissolved-oxygen level of its outflow at 8.0 mg/L or above. An inspector takes a random sample of 100 readings and finds $\bar{x} = 7.62$ mg/L and $s = 1.9$ mg/L. The inspector tests $H_0 : \mu = 8.0$ against $H_a : \mu < 8.0$. Use $P(Z < -2.00) = 0.0228$.

**a)** Check the conditions, compute the test statistic and the $p$-value.

**b)** State the decision and the conclusion in context at $\alpha = 0.01$, and then at $\alpha = 0.05$.

**c)** For each of the two decisions in (b), name the type of error that could have been made, and describe it in context.